Linearization in elastodynamics

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Linear elasticity is a well-established and probably constitutes thes most used modeling approach in engineering applications of solid mechanics. Going back to Dal Maso, Negri and Percivale, it can be rigorously derived via a Γ Limit procedure from nonlinear models. As this procedure is purely variational, it does not directly translate to force balance models like elastodynamics. Nonetheless using an approximation procedure due to Benešová, Kampschulte and Schwarzacher, we explore the connection between the Γ Limit and elastodynamics.

This is a joint work with Malte Kampschulte and Martin Kružík.

On the role of quasiconvexity in dynamics

Konstantinos Koumatos
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Quasiconvexity is known to be the central notion of convexity in the vectorial calculus of variations as, for example, quasiconvexity of the energy density is equivalent to the lower semicontinuity of the energy functional in weak Sobolev topologies, guaranteeing the existence of minimisers via the direct method. Similarly, quasiconvexity appears as part of the necessary and sufficient conditions for a map to be a strong local minimiser. However, in dynamics, the role of quasiconvexity has not been equally understood. In this talk, I will present some results where quasiconvexity of the entropy restores weak strong uniqueness for general systems of conservation laws endowed an involution. Examples of these systems include the equations of isothermal elasticity as well as adiabatic thermoelasticity. These results are achieved by establishing an appropriate Garding-type inequality for the quantity known as relative entropy in the theory of conservation laws, akin to the Weierstrass excess function in the calculus of variations. This is a series of works joint with Stefano Spirito (University of L'Aquila), Myrto Galanopoulou (University of Sussex), and Andreas Vikelis (Ricardo Plc).

Elastic materials with microscopic, elastically rigid magnetic inclusions

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Abstract

We will discuss a variational model for a heterogeneous, linearly elastic magnetic material that incorporates elastically rigid magnetic inclusions firmly bonded to the matrix. Unlike in the perhaps more traditional micromagnetic model of Brown, the magnetic state is described using only the magnetic field, instead of the magnetization. As the main result, we obtain an effective magnetoelastic energy, by periodic homogenization as the period approaches zero.

Joint work: **Raffaele Grande** (Duisburg-Essen), **Martin Kružík** (UTIA Prague), **Giuseppe Tomassetti** (Rome 3)

Positive temperature in nonlinear thermoviscoelasticity

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According to the third law of thermodynamics, the absolute zero temperature cannot be attained. Starting from a positive initial temperature, we prove the existence of solutions to a Kelvin–Voigt model for quasi-static, nonlinear thermoviscoelasticity in a finite-strain regime, with the temperature bounded from below by an exponentially decaying function of time. We subsequently consider small perturbations around the identity deformation and temperatures near a critical positive reference value. In this regime, we show that weak solutions of the nonlinear system converge, in a suitable sense, to solutions of a linearized thermoviscoelastic model. Additionally, we present several computational experiments that illustrate the theoretical findings. This research is conducted in collaboration with R. Badal (Erlangen), M. Friedrich (Erlangen), L. Machill (Bonn), and M. Horák (Prague).

Towards a 1d model for tape spring devices

Roberto Paroni
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Tape springs, which are slender metallic strips featuring a pre-curved cross-section, serve as an appealing structural option and hinge mechanism for deployable structures due to their lightweight design, affordability, and overall simplicity. For these reasons, the study of these devices has been particularly prolific in recent years. In this talk, we model tape spring devices using shell theory and analyze their asymptotic behavior as the width of the cross-section approaches zero.
The talk is based on ongoing work with Marco Picchi-Scardaoni.

Mathematical aspects of the equations of viscoelasticity with strain gradients

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We study the mathematical theory of models that combine viscoelastic effects of Kelvin-Voigt type with effects of higher strain gradients and nonconvex energies. One can show that in a certain parameter regime the resulting system is equivalent to a full viscosity approximation of the equations of elasticity with nonconvex energies. I will review existence results and asymptotic limits as the viscosity or or as the characteristic length the effect of strain gradients goes to zero. In particular, this provides an existence theory for a simple model of elasticity with higher gradients.

The second part of the talk is devoted to the viscosity-capillarity approximations of the elasticity equations and the equations of quantum hydrodynamics with artificial viscosity. The goal is to test the combined effect of diffusion and dispersion when the limit is a hyperbolic system. We discuss thresholds for convergence in one-space dimension when both viscosity and capillarity tend to zero. Moreover, we show that the combined effect of viscosity and capillarity can produce traveling waves with oscillatory tails.